Dynamics 4

Learning Module 1

Lagrange's method for conservative systems

THE UNIVERSITY *of* EDINBURGH
School of Engineering

DYNAMICS 4

LM1 GUIDE 26 27

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Learning Module 1 — Lagrange's method for conservative systems

Estimated time: 20 hours Complete module before: 6 October

1. Start by reading the Course Notes: for this module you should read **Sections 2.1 and 2.2.** Take hand-written notes as you read.

2. Attend the lectures and tutorial sessions and, if you need, watch the lecture recordings later. This will help consolidate what you learned. Keep taking notes.

3. Attempt to solve **Questions 1.1 to 1.4** in **Exercise Sheet 1**. Compare your solutions to the numerical solutions provided at the end of each question. Write down any questions you might have.

4. Compare your solutions to the worked-out example solutions to **Questions 1.1 to 1.4**. Discuss any questions you might have in the tutorial sessions. Arrange to meet with your lecturer if you need further clarifications.

5. Be active in the Learn discussion board for this Learning Module, which is labeled **analytical_dynamics**. You can post your own questions and help clarify or answer the questions of others.

6. Have a look at the additional resources provided in the **Learning Module 1** folder on Learn. You may also want to read **Section 2.6** of the Course Notes with examples of real life applications.



In this module

An introduction to analytical dynamics
Generalised coordinates
Lagrange's formalism for conservative systems
Kinetic and potential energy in dynamic systems
Prescribed boundary motion
Non-linear dynamic behaviour

Learning outcomes

Apply virtual work-based methods to dynamical systems, relating between Lagrangian and Newtonian mechanics

Derive equations of motion for dynamic systems from energy-based approaches (e.g. Lagrange's method)

This is a study and learning guide to Learning Module 1. You can find the corresponding lecture recordings and slides on Learn. This module has a dedicated thread in the discussion board on Learn.

Please note that the course time and the recommended *before dates*, are estimated and may differ from person to person. You should not see these as targets but instead as a way to help you plan your semester and the preparation for exams. The estimated time includes going through all the course materials, interaction in the discussion boards and self-study time.

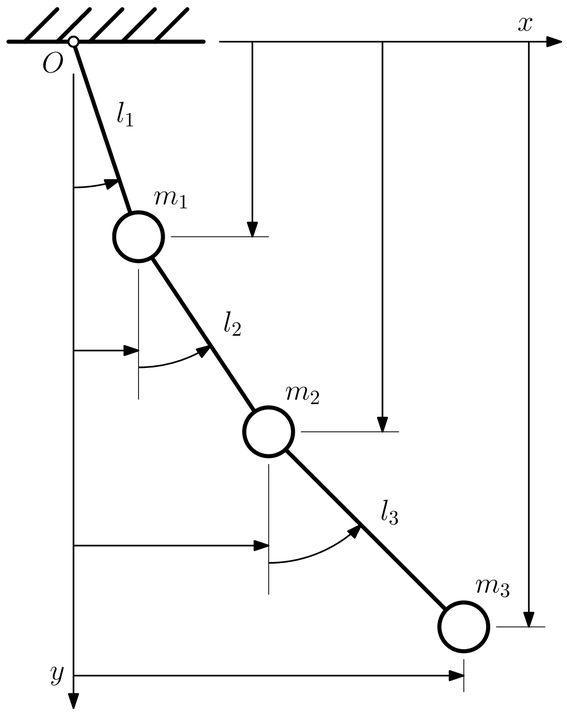
Generalised coordinates

- Multi-degree of freedom (d.o.f.) systems



Generalised coordinates

- Multi-degree of freedom (d.o.f.) systems



6 $(x_j,\ y_j) \quad j = 1,\ 2,\ 3$

↓

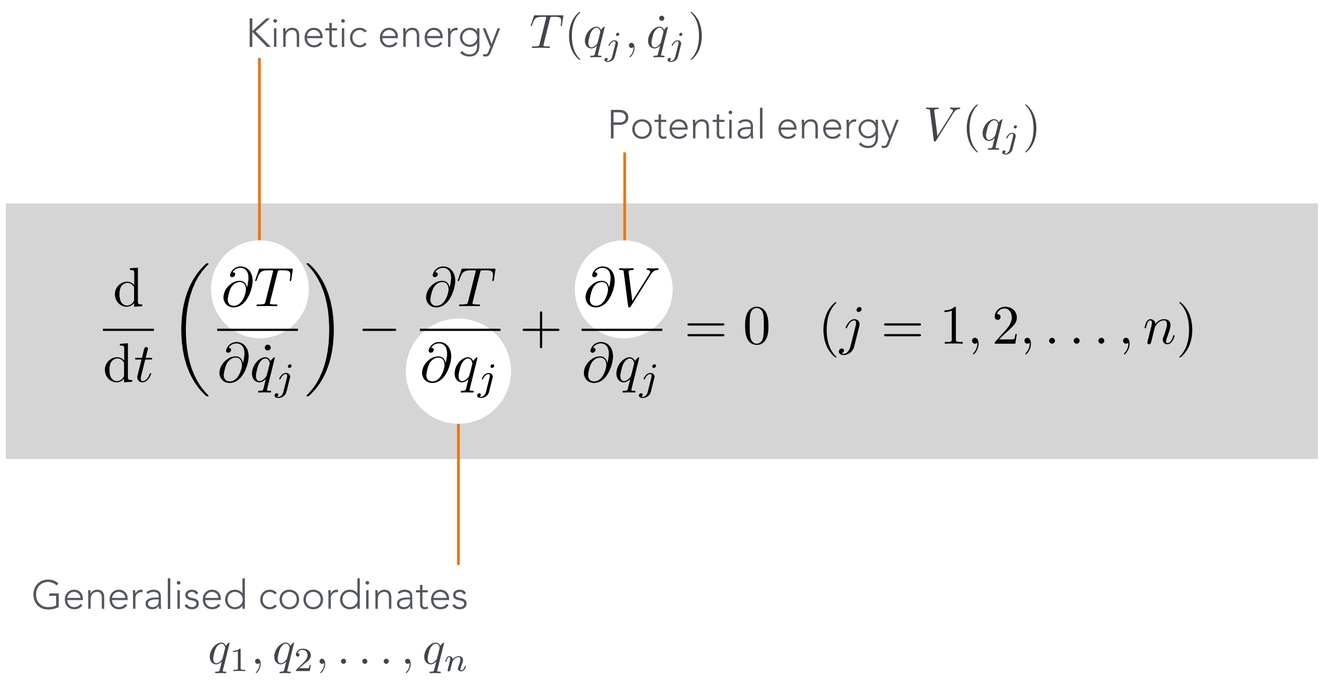
Not independent (constrained)

↓

3 $\varphi_j \quad j = 1,\ 2,\ 3$

Lagrange's equations

- Conservative systems (energy does not change)
- i.e. no friction/damping and no external forces doing work
- For an *n*-degree-of-freedom (*n*-d.o.f.) system:

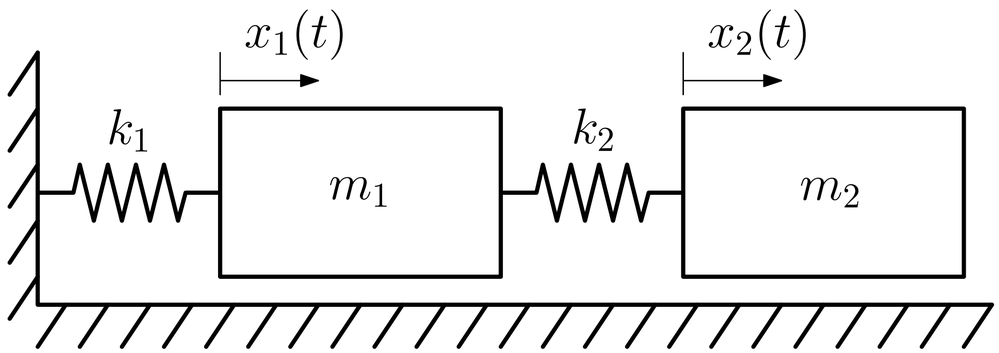


Example (CN)

Determine the equations of motion for the system shown

a) Using free body diagrams (FBD).

b) Using Lagrange's equations.



Setup Lagrange's equations

Determine correct expressions for:

a) Kinetic energy

b) Potential energy

Setup Lagrange's equations

Determine correct expressions for:

a) Kinetic energy

- Linear motion (pure): $T_{\mathrm{t}} = \frac{1}{2} M v^2$
- Angular motion (pure): $T_{\mathrm{r}} = \frac{1}{2} I_A \dot{\varphi}^2$
- General plane motion: $T_{\mathrm{p}} = \frac{1}{2} \left(M v_{\mathrm{G}}^2 + I_{\mathrm{G}} \dot{\varphi}^2 \right)$

Note: $v = ||\vec{v}|| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

Setup Lagrange's equations

Determine correct expressions for:

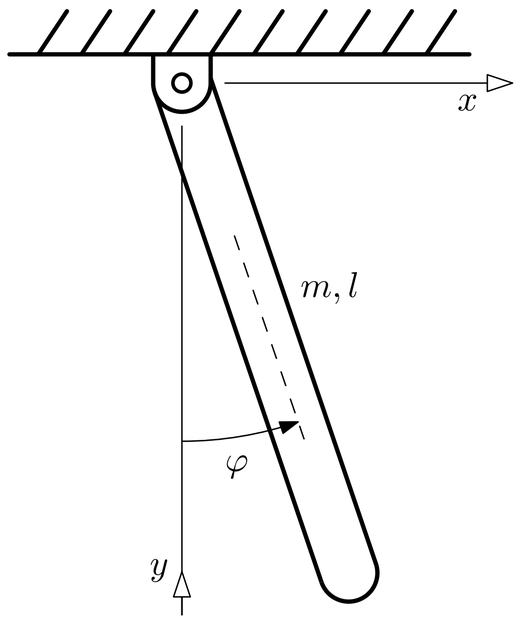
b) Potential energy

- Gravitational: $V_{\mathrm{g}} = Mgh$
- Elastic, linear (spring): $V_{\mathrm{s}} = \frac{1}{2} k \Delta x^2$
- Elastic, torsion (twist): $V_{\mathrm{t}} = \frac{1}{2} K_{\mathrm{T}} \alpha^2$

with $K_{\mathrm{T}} = \frac{G \pi d^4}{32 L}$

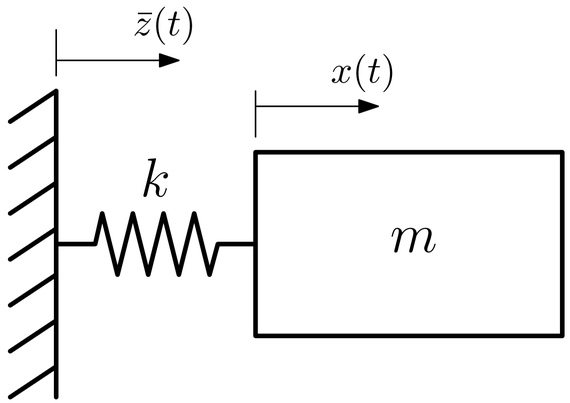
Example (CN)

Formulate the equations of motion for the system shown using Lagrange's equations.



Systems with prescribed boundary motions

- Vibration transmission
- Ground motion (e.g. earthquakes)



Systems with prescribed boundary motions

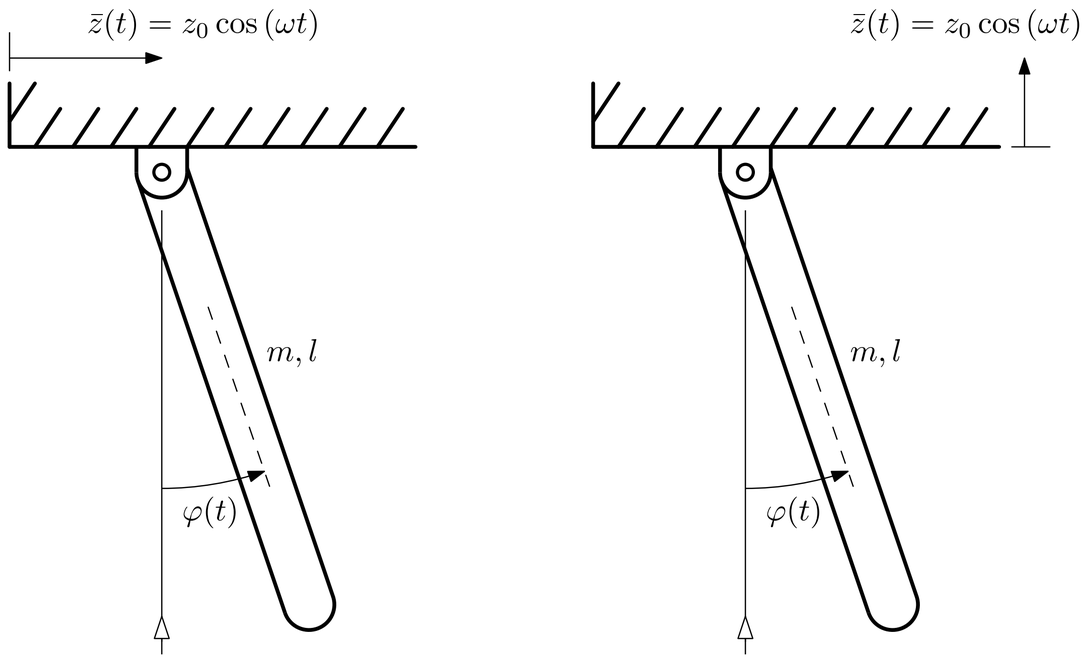
- The moving boundary coordinate
  - is not a force
  - is not a generalised coordinate
  - ... it is a constraint

- How to deal with such systems?

  ... include boundary motion in energy expressions

Example (CN)

Find the equations of motion for the systems shown below using Lagrange's equations.



Dynamic systems with non-linear response

- Equations of motion with non-linear terms:
  - Not proportional
  - Higher powers
  - Products
  - Non-linear functions (e.g. sin, cos)
  - …

Dynamic systems with non-linear response

- Linear systems
  - Easy to analyse
  - Have explicit solutions
  - Superposition is valid
  - Solutions are proportional to stimuli
  - Possible physical behaviour completely known

Dynamic systems with non-linear response

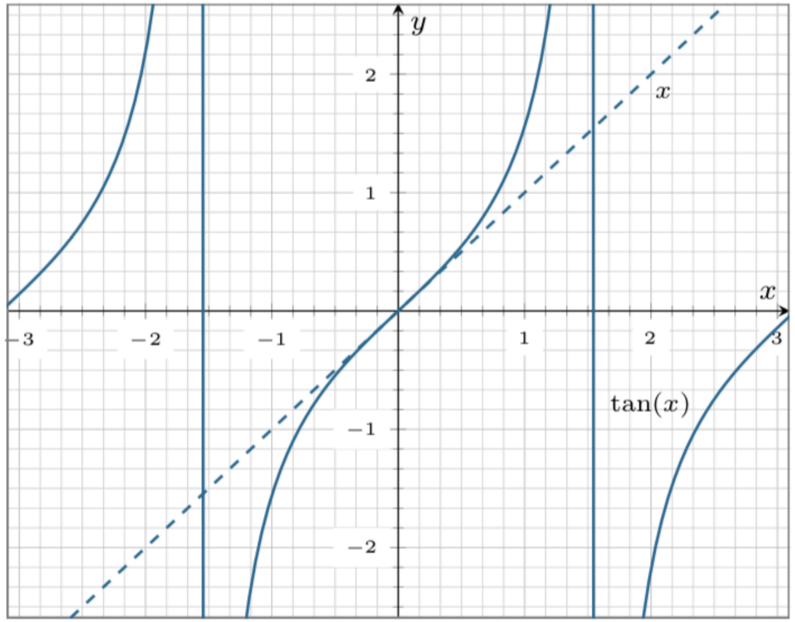
- Non-linear systems
  - No exact solutions (numerical solution often needed)
  - No superposition
  - Solutions not proportional to stimuli
  - Complex
  - Physical behaviour not fully known

Dynamic systems with non-linear response

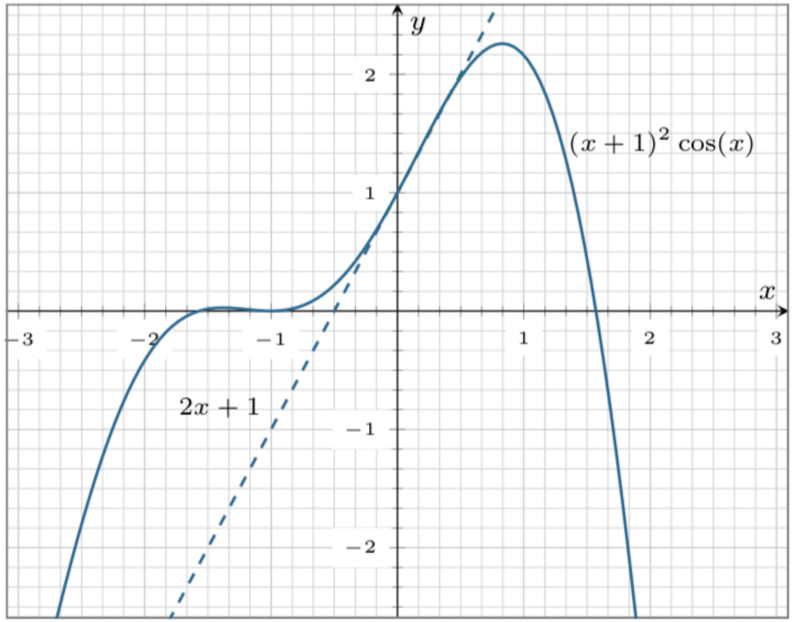
- What to do?
  - ...
  - Find approximate solution (not always reasonable)
  - Approximate non-linear to linear
    1. Find reference point (e.g. equilibrium configuration)
    2. Assume small vibrations (!)
    3. Replace all non-linear functions (or terms) by their first order expansions

Dynamic systems with non-linear response

- Non-linear to linear:



$\tan(x) \approx x$



$(x+1)^2\cos(x) \approx 2x+1$

Dynamic systems with non-linear response

- Non-linear to linear:

$$\cos(x) \approx \cosh(x) \approx 1$$

$$\cos(x + \pi) \approx -1$$

$$\cos(x + \frac{\pi}{2}) \approx -x$$

$$\cos(x - \frac{\pi}{2}) \approx x$$

$$\sin(x) \approx \sinh(x) \approx x$$

$$\sin(x + \pi) \approx \sin(x - \pi) \approx -x$$

$$\exp(x) \approx 1 + x$$

(...)

Example (CN)

Determine the equations of motion for the system shown using Lagrange's equations.

